Budget Feasible Roadside Unit Allocation Mechanism in Vehicular Ad-Hoc Networks

Xiaohua Xu*, Shuibing He†, Meng Han*, Reza M. Parizi*, Gautam Srivastava‡

* College of Computing and Software Engineering, Kennesaw State University, Marietta, GA 30060, USA
† School of Computer Science and Technology, Zhejiang University, Hangzhou, China
‡ Department of Mathematics and Computer Science, Brandon University, Brandon, MB, Canada
Email: xxu6@kennesaw.edu, heshuibing@zju.edu.cn, mhan9@kennesaw.edu, rparizi1@kennesaw.edu, srivastavag@brandonu.ca

***Abstract*—The Roadside Unit (RSU) allocation is critical for the functionality and topology control of Vehicular Ad-Hoc Networks. However, due to the complexity of different transportation scenarios and the challenging coordination among different RSUs, the allocation is still a challenging issue in both the academic and practical industry. In this paper, we utilize the *game theoretic RSU deployment* to fundamentally improve the allocation of RSUs with practical consideration. Given a set of RSUs of arbitrary covering radii, assuming there is a budget requirement that specifies the total number of RSUs to be placed. In addition, considering the minimum distance requirement between any pair of RSUs, how to select a subset of RSUs to cover the maximum number of Points of Interest (POIs). We consider the selfish behaviors of RSU allocation and apply a *game theoretic* technique. We propose a mechanism to achieve a small price of anarchy.**

***Keywords*—Adhoc; Networks; Game theory; Feasibility; Allocation; Roadside Unit, Vehicular Networks**

I. Introduction

With the development of electric vehicles and "smart" cars, vehicular Ad-Hoc networks increasingly occupy an important place in the automotive world. It is indicated that 100% of new cars in the U.S./ E.U. / China will become "connected" by 2022 because of legal and customer pull for connected cars. In terms of predictions, by the end of 2030, electric car sales will account for 55% of all new car sales. During this transformation period, new car sales in Europe could increase by 34%, from approximately 18 million to just over 24 million USD. While in the U.S.A., there could be a growth of 20% (22 million units) as PricewaterhouseCoopers (PwC) Autofacts team assumes [1].

Along with the development of connected vehicles, vehicular networks also need a lot of effort to be integrated with more technologies and advanced features. One of the big challenges of vehicular Ad-hoc networks is how to arrange the RSU with the network to coordinate the whole system well [2], [3]. Dedicated Short Range Communications (DSRC) contains two units that have transceivers and transponders: RoadSide Units(RSUs) and On-Board Units(OBUs). DSRC is also a secure and consistent form of communication between a vehicle and the roadside in specific positions since it operates around 5.9 GHz frequency band. As is shown in Fig. 1, the RSUs engage traveling vehicles via the DSRC technology aim to reach the essential traffic information such as time, speed and location of the vehicle.

In this paper, we model the RSU selection and allocation as a game-theoretical model. The motivation of using mechanism design for the problem is as follows. In practice for mechanism design, the true valuation v_i is unknown, in addition, each RSU candidate's bidding value may be different from its valuation. Considering the selfish behavior of each RSU candidate, we provide a method to efficiently and effectively optimize the whole network with the help of our proposed algorithms. Suppose there is a set of players, each has an RSU candidate position. Suppose all players interact to select an outcome. Assume each player has a valuation indicating the number of POIs covered and bid for his request. The objective of this mechanism is to compute an allocation algorithm and payment rules. Its performance is appraised by the *Price of Anarchy(PoA)*, which consider a trade-off between the social welfare and the optimized mixed Nash equilibrium. And this work aims to design and create an effective mechanism with the minimum of PoA. As far as our knowledge goes, no such mechanism design for RSU placements has been studied with a small price of anarchy for Nash equilibrium.

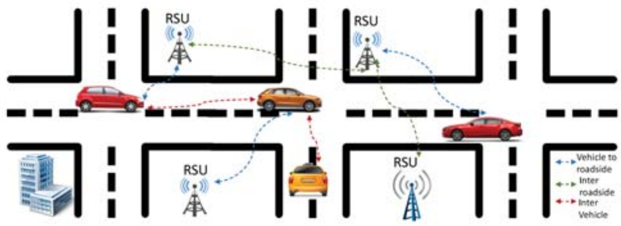

Fig. 1. Model of RSU Communications

Our Contributions: For the RSU allocation, we apply a *game theoretic* technique and present a mechanism to solve this problem.

- We design an efficient Linear Programming (LP) based algorithm for RSU allocation. The algorithm follows the relax-and-round scheme.
- Considering the algorithmic results of the problem, we introduce this mechanism on the grounds of the LP relaxation and prove the smoothness of this mechanism.
- We further propose a mechanism based on the relax-and-round scheme. We have proved that the PoA is bounded by a small number which only depends on the local independence number.

The rest of the paper is organized as follows. Section II formulates the mechanism design problem. Section III presents a RSU allocation algorithm. Section IV presents a LP-relaxation based mechanism. Section V presents a relax-and-round mechanism. Section VI presents the numerical results.

978-1-7281-5207-3/20/$31.00 ©2020 IEEE

Section VII reviews the related work. Section VIII concludes the paper.

II. System Model

Suppose there is a set of players $N = \{1, \ldots, n\}$ in a two-dimensional plane. Each player i is associated with a location and send a RSU allocation request here is a minimum distance requirement between any pair of deployed RSUs. The total number of RSUs to be placed is at most K. The set of all RSU allocation requests form a set of RSU candidates.

Suppose all players interact to select an outcome. Let Ω represent the set of all feasible outcomes. Assume each player i has a valuation function $v_i \colon \Omega \to \mathbb{R}_{\geq 0}$ which maps each outcome to a real value. We use v for the evaluation profile that specifies a evaluation for each player.

Assume each player needs to bid for his request. We use b for the bid profile that specifies a bid for each player. Based on b, the central auctioneer uses an allocation algorithm f to compute an outcome $f(b) \in \Omega$. Based on allocation algorithm, central auctioneer also uses a payment rule p to compute the payments $p(b) \in \mathbb{R}_{\geq 0}$ of all bidding players. Both the allocation algorithm and the payment rule can be randomized.

Assume each player pays what they have bid if his request is satisfied, *i.e.*, $p_i(b) = b_i(f_i(b))$. We assume that player i's utility is $u_i(b, v_i) = \mathbf{E}v_i(f(b)) - \mathbf{E}p_i(b)$. The valuation of each of players are aware. A potentially randomized bid profile b that may depend on v is a *mixed Nash equilibrium* if for each player $i \in N$ and possible deviations b'_i that may depend on v, we have $\mathbf{E}_b[u_i(b, v_i)] \geq \mathbf{E}_{b'_i, b_{-i}}[u_i((b'_i, b_{-i}), v_i)]$.

We evaluate the performance of mechanisms by their *Price of Anarchy (PoA)*. Define $B_{\mathsf{NASH}}(v)$ as the set of all mixed Nash. Then,

$$PoA = \max_{v} \max_{b \in B_{\mathsf{NASH}}(v)} \frac{\max_{x \in \Omega} \sum_{i \in N} v_i(x)}{\mathbf{E} \sum_{i \in N} v_i(f(b))}$$

The objective is to minimize the PoA.

Smoothness Framework: A mechanism is (λ, μ)-*smooth* [4], [5] for $\lambda, \mu \geq 0$ if for all valuation profiles v and all bid profiles b there exists a possibly randomized strategy b'_i for each player i that may rely upon the valuation profile v of all players and the bid b_i of that player such that

$$\sum_{i \in N} \mathbf{E}u_i((b'_i, b_{-i}), v_i) \geq$$
$$\lambda \cdot \max_{x \in \Omega} \sum_{i \in N} v_i(x) - \mu \cdot \sum_{i \in N} \mathbf{E}p_i(b).$$

Relax-and-round algorithms: For any optimization problem, a potentially randomized algorithm A receives the functions w as input and calculate an output $A(w) \in \Omega$ [5]. The algorithm is an α-*approximation algorithm*, for $\alpha \geq 1$, if for all weights w, $\mathbf{E}w(A(w)) \geq \frac{1}{\alpha} \cdot \max_{x \in \Omega} w(x)$. A rounding algorithm is *oblivious* if it does not require knowledge of the actual objective function w, beyond the fact that x was optimized with respect to w. Formally, a rounding scheme is an α-*approximate oblivious rounding scheme* if it computes a solution x such that for all w, $\mathbf{E}w(x) \geq \frac{1}{\alpha} w(x')$.

As there is a minimum distance requirement between any pair of deployed RSUs, let the neighborhood $N[i]$ denotes all the RSU candidates that conflict with i, including i itself.

$$N[i] = \{j : (i, j) \in E\} \cup \{i\}.$$

Let η be the maximum number of RSU candidates in $N[i]$ for any possible i which are not conflict with each other. For each RSU candidate i, let $N[N[i]]$ be the set of all RSU candidates within two hops of the RSU candidate i, *i.e.*, $j \in N[N[i]]$ if and only if $\exists k \in N[i]$ such that $(j, k) \in E$. Note that here $N[i] \subseteq N[N[i]]$. Let η_2 be the maximum number of independent RSU candidates in $N[N[i]]$ for any possible i.

III. Relax-and-Round Algorithm

Maximizing the social welfare is the goal of the RSU allocation. First, we formulate the problem as an integer linear program (ILP). We then make a LP relaxation. After that, we build a RSU allocation algorithm from an approximate oblivious rounding of the solution to the LP relaxation.

A. LP Relaxation

Let us first define the ILP. Let a binary variable $x_i \in \{0, 1\} : i \in [n]$ denote whether a RSU candidate i is selected. As the valuation of RSU candidate i is v_i, the objective aims to find $\max \sum_{i \in N} v_i \cdot x_i$ subject to three types of constraints. The first type of constraints is that, for any pair of conflicting players i and j, we have $x_i + x_j \leq 1$. The second one is that, considering the neighborhood $N[i]$ of each player i, by the definition of local independence number, we have $x(N[i]) \leq \eta, \forall i$. The third one is that the total number of RSUs selected is at most K, thus we have $\sum x_i \leq K$. To sum up, we have the ILP given in Equation 1.

$$\begin{aligned} &\text{Max} \quad \sum v_i \cdot x_i \\ &\text{s.t.} \\ &\begin{cases} x_i + x_j \leq 1, \forall (i, j) \in E \\ x(N[i]) \leq \eta, \forall i \in [n] \\ \sum x_i \leq K \\ x_i \in \{0, 1\}, \forall i \in [n] \end{cases} \end{aligned} \tag{1}$$

If we allow the value of x_i to be fractional, we have the LP relaxation of Equation (1), shown in Equation 2.

$$\begin{aligned} &\text{Max} \quad \sum v_i \cdot x_i \\ &\text{s.t.} \\ &\begin{cases} x_i + x_j \leq 1, \forall (i, j) \in E \\ x(N[i]) \leq \eta, \forall i \in [n] \\ \sum x_i \leq K \\ x_i \geq 0, \forall i \in [n] \end{cases} \end{aligned} \tag{2}$$

In the LP relaxation, the binary variables $x_i \in \{0, 1\}$ are substituted by non-negative variables $x_i \geq 0$. It could be interpreted that x_i is a fractional allocation to RSU candidate i. As $x_i + x_j \leq 1, \forall (i, j) \in E$, we have $x_i \leq 1, \forall i \in [n]$.

Input : $v_i : \forall i \in [n]$
Output: S''

Accomplish an optimal fractional solution by solving LP relaxation x;
Let $\alpha = 2$;
Sample each RSU candidate independently with probability $\frac{x_i}{\alpha\eta}$;
Let S denote the set of chosen RSU candidates;
We call an RSU candidate in S an S-candidate;
We process S-candidate in increasing order of their covering radii;
For each RSU candidate i, mark i for any constraint $x_i + x_j \leq 1, \forall(i,j) \in E$ if some other S-candidate $j < i$ appears in this constraint;
Remove all marked RSU candidates, and return S', the set of remaining RSU candidates;
If $S' \leq K$, randomly select K RSU candidates, and return S";
return S''

Algorithm 1: Relax-and-Round Algorithm for RSU allocation

B. Rounding Algorithm for RSU Allocation

After solving this LP relaxation, we apply oblivious rounding. The main idea is to select randomly a subset of RSU candidates based on x_i. Then we remove all RSU if it conflicts with any of the three types of constraints. The algorithm is shown in Algorithm 1.

C. Performance Analysis

We then analyze the performance of this randomized algorithm.

Lemma 1. *Solution S" is feasible for the RSU allocation problem.*

Proof: The algorithm guarantees that at most one RSU candidate in S' participates in any conflicting constraint. This means that for any pair of conflicting RSU candidates, at most one RSU candidate is selected to S'. In addition, at most K RSU candidates are finally selected to S". ■

Next, we prove the following result.

Lemma 2. *For any RSU candidate i, $Pr[i \in S'|i \in S] \geq 1 - \frac{1}{\alpha}$.*

Proof: For any RSU candidate i, for any constraint c in the form of $x_i + x_j \leq 1$, we compute the probability that i is marked for deletion. Since i is marked only if $j \in S$, we have:

$$Pr[i \text{ is marked for constraint } c|i \in S] \leq \frac{x_j}{\alpha\eta} \tag{3}$$

for any RSU candidate i, take the union bound over all constraints of the form $x_i + x_j \leq 1, \forall(i,j) \in E$.

$$Pr[i \text{ is marked}|i \in S] \leq \sum_{j \in N[i]} \frac{x_j}{\alpha\eta} \leq \frac{1}{\alpha}. \tag{4}$$

Thus, the probability that i is deleted from S conditional on it being chosen in S is at most $\frac{1}{\alpha}$. We have

$$Pr[i \in S'|i \in S] \geq 1 - \frac{1}{\alpha}. \tag{5}$$

■

Lemma 3. *For any RSU candidate i, $Pr[i \in S"|i \in S'] \geq \frac{1}{\alpha\eta}$.*

Proof: As $|S'| \leq |S| \leq \alpha\eta$, and we select K out of $|S'|$ RSU candidates, the theorem follows. ■

Finally, we prove the main result. Observe that our algorithm always outputs a workable resolution. To prove the lower bound of the objective value, recall that $Pr[i \in S] = \frac{x_i}{\alpha\eta}$ for all i. Thus, we have

$$Pr[i \in S\backslash] \geq Pr[i \in S] \cdot Pr[i \in S'|i \in S]] \cdot Pr[i \in S"|i \in S`] \tag{6}$$

$$\geq \frac{x_i}{\alpha\eta} \cdot (1 - \frac{1}{\alpha}) \cdot \frac{1}{\alpha\eta} \tag{7}$$

Finally, we obtain Theorem 1.

Theorem 1. *There is a randomized $8\eta^2$-approximation oblivious rounding algorithm for the RSU allocation problem.*

Proof: The proof is based on the linearity of expectation and $\alpha = 2$. ■

IV. MECHANISM FOR LP RELAXATION

Previously, given the input of each RSU candidate i's valuation v_i of being selected, we solve both (1) LP relaxation and (2) propose a RSU allocation algorithm to maximize the social welfare. In this section, we propose a mechanism for LP relaxation correspondingly. In the next section, we will propose a mechanism for relax and round algorithm.

A. Mechanism for LP Relaxation

The mechanism for LP relaxation is as follows.

- each RSU candidate i submits a bid b_i.
- the central authority collects all bids and use the vector b as the input for LP relaxation and solve the LP relaxation.
- if a RSU candidate i is allocated with x_i, it will be charged $b_i \cdot x_i$ for being selected.

B. PoA Upper Bound

We will show in Lemma 5 that this mechanism for the LP relaxation of a RSU allocation (Equation 2) is $(1/2, \eta_2 + 1)$-smooth for deviations to $b'_i = \frac{1}{2}v_i$. The claimed bound on the POA then follows from Theorem 2.

To prove Lemma 5, given a bid vector b , we denote by W^b the value of the optimal LP solution. Given a bid vector b and a value of $\overline{x_i}$, we denote by $W^b(i, \overline{x_i})$ the optimal social welfare of all other agents when the i-th agent has been allocated a fraction of $\overline{x_i}$, *i.e.*, $W^b(i, \overline{x_i})$ is the optimal value of the following LP.

$$
\begin{aligned}
&\text{Max} \quad \textstyle\sum_{j \neq i} b_j \cdot x_j \\
&\text{s.t.} \\
&\begin{cases}
x_j + x_k \leq 1, \forall (j,k) \in E \\
x(N[j]) \leq \eta, \forall j \in [n] \\
\sum x_i \leq K \\
x_j \geq 0, \forall j \neq i, j \in [n] \\
x_i = \overline{x_i}
\end{cases}
\end{aligned} \tag{8}
$$

We use v_{-i} to denote the valuations of the players other than i. Let $(0, b_{-i})$ denotes the bidding vector obtained by setting the i-th entry of the vector b to be zero.

Lemma 4. *Let $\bar{x}$ be an arbitrary fractional solution. Then,*

$$\sum_{i \in N} \left(W^{(0,b_{-i})} - W^{b_{-i}}(i, \overline{x}_i) \right) \leq (\eta_2 + 2) \cdot W^b.$$

Proof: Let $\hat{x}$ denote an optimal solution corresponding to W^b. Based on $\hat{x}$, we define a LP solution $\hat{x}^{-i}$ by setting

$$\hat{x}_j^{-i} = \begin{cases} (1 - \overline{x}_i)\, \hat{x}_j, \text{if } j \in N[N[i]] \\ (1 - \frac{\overline{x}_i}{K})\, \hat{x}_j, \text{else} \end{cases} \tag{9}$$

We next verify that whether $\hat{x}^{-i}$ is a feasible to $W^{b_{-i}}(i, \overline{x}_i)$. We only need to verify the constraints that contain x_i.

Case 1: for each constraint of the form $x_i + x_j \leq 1$, as $\hat{x}_j^{-i} = (1 - \overline{x}_i)\, \hat{x}_j$, thus $\hat{x}_j^{-i} + \overline{x}_i \leq 1 \leq (1 - \overline{x}_i) + \overline{x}_i = 1$, the constraint is satisfied.

Case 2: for the constraint $x(N[i]) \leq \eta$, as $\forall j \in N[i]$, we have $\hat{x}_j^{-i} = (1 - \overline{x}_i)\, \hat{x}_j$, this constraint is also satisfied.

Case 3: for the constraint of the form $x(N[j]) \leq \eta$ such that $i \in N[j]$; for each $k \in N[j]$, since $(i,j) \in E$ and $(j,k) \in E$, we have $k \in N[N[i]]$; We have $\hat{x}_k^{-i} = (1 - \overline{x}_i)\, \hat{x}_k$. Thus, $x(N[j]) \leq \overline{x}_i + \sum_{k \in N[j], k \neq i} (1 - \overline{x}_i)\, \hat{x}_k \leq (1 - \overline{x}_i)\, \eta$ this constraint is satisfied. Thus, $\hat{x}^{-i}$ is feasible.

Case 4: for the constraint of the form $\sum x_i \leq K$, We have $\hat{x}_k^{-i} = (1 - \overline{x}_i)\, \hat{x}_k$. Thus, $\sum \hat{x}_j^{-i} = \sum_{j \neq i} \hat{x}_j \cdot \frac{K - x_i}{K} + x_i \leq K$. this constraint is satisfied.

To sum up, $\hat{x}^{-i}$ is feasible. Finally, we have

$$
\begin{aligned}
&\sum_{i \in N} W^{b_{-i}}(i, \overline{x}_i) \geq \sum_{i \in N} \sum_{j \neq i, j \in N} b_j \hat{x}_j^{-i} \\
&= \sum_{j \in N} \sum_{i \neq j, i \in N} b_j\, (1 - \overline{x}_i)\, \hat{x}_j \\
&= \sum_{j \in N} b_j \hat{x}_j \sum_{i \neq j, i \in N} (1 - \overline{x}_i) \\
&= \sum_{j \in N} b_j \hat{x}_j (n - 1 - \sum_{i \neq j, i \in N} \overline{x}_i) \\
&= (n-1) \sum_{j \in N} b_j \hat{x}_j - \sum_{j \in N} b_j \hat{x}_j \cdot \sum_{i \neq j, i \in N} \overline{x}_i \\
&\geq (n - \eta_2 - 2) \cdot \sum_{j \in N} b_j \hat{x}_j \\
&= (n - \eta_2 - 2) \cdot W^b,
\end{aligned}
$$

which gives the claimed bound as clearly $W^{(0,b_{-i})} \leq W^{(b_i,b_{-i})} = W^b$. ∎

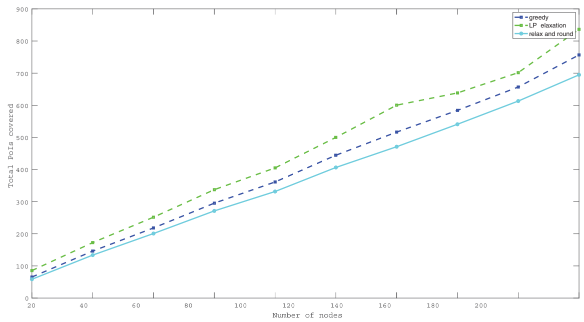

Fig. 2. The total PoIs covered when the density of RSU players is fixed.

Lemma 5. *The pay-your-bid mechanism that solves the LP relaxation in Equations (2) is $(1/2, \eta_2 + 2)$-smooth for deviations to $b_i' = \frac{1}{2} v_i$.*

Proof: Consider valuations v, bids b and deviations of each player $i \in N$ to $b_i' = 1/2 \cdot v_i$. Denote the optimal fractional allocation for bids (b_i', b_{-i}) by $\bar{x}_1(b_i', b_{-i}), \ldots, \bar{x}_n(b_i', b_{-i})$. Then, by the definition of b_i',

$$
\begin{aligned}
&u_i((b_i', b_{-i}), v_i) = v_i(\bar{x}_i(b_i', b_{-i})) - b_i'(\bar{x}_i(b_i', b_{-i})) \\
&= b_i'(\bar{x}_i(b_i', b_{-i})).
\end{aligned}
$$

Since $\bar{x}_1(b_i', b_{-i}), \ldots, \bar{x}_n(b_i', b_{-i})$ is fractional allocation that maximizes declared welfare with respect to bids (b_i', b_{-i}),

$$
\begin{aligned}
&b_i'(\bar{x}_i(b_i', b_{-i})) + W^{b_{-i}} \\
&\geq b_i'(\bar{x}_i(b_i', b_{-i})) + \sum_{j \neq i} b_j(\bar{x}_j(b_i', b_{-i})) \\
&\geq b_i'(\bar{x}_i(v)) + W^{b_{-i}}(i, \bar{x}_i(v))
\end{aligned}
$$

By reorganizing this, it displays

$$b_i'(\bar{x}_i(b_i', b_{-i})) \geq b_i'(\bar{x}_i(v)) - [W^{b_{-i}} - W^{b_{-i}}(i, \bar{x}_i(v))].$$

After adding all players together and applying Lemma 4, we could obtain

$$
\begin{aligned}
&\sum_{i \in N} u_i((b_i', b_{-i}), v_i) = \sum_{i \in N} b_i'(\bar{x}_i(b_i', b_{-i})) \\
&\geq \sum_{i \in N} \left(b_i'(\bar{x}_i(v)) - [W^{b_{-i}} - W^{b_{-i}}(i, \bar{x}_i(v))] \right) \\
&\geq \sum_{i \in N} b_i'(\bar{x}_i(v)) - (\eta_2 + 2) \cdot \sum_{i \in N} b_i(\bar{x}_i(b)) \\
&= \frac{1}{2} \cdot \sum_{i \in N} v_i(\bar{x}_i(v)) - (\eta_2 + 2) \cdot \sum_{i \in N} b_i(\bar{x}_i(b)),
\end{aligned}
$$

which completes the proof. ∎

V. Mechanism for Relax-and-Round Algorithm

A. Converting relax-and-round algorithm to Mechanism

The mechanism is based on the proposed RSU algorithm for the allocation of the requests and compute the PoA.

- each RSU candidate i submits a bid b_i.
- the central authority collects all bids and use the vector b as the input and run the Algorithm 1.
- if a RSU candidate i is selected, it will be charged b_i for being selected.

B. PoA Upper Bound

Theorem 2. *[5] Suppose the $M' = (f', p')$ mechanism could work for the optimization of Π' relaxation with (λ, μ)-smooth with the deviations of $b'_i = \frac{1}{2}v_i$, then for Π, which is gained the $(\lambda/\alpha, \mu)$-smooth relaxation through an α-approximate based on the pay-your-bid mechanism $M = (f, p)$.*

Finally, the following theorem establish a connection between smoothness and PoA.

Theorem 3 (Syrgkanis and Tardos [4])**.** *If a mechanism is (λ, μ)-smooth and it is possible for players to withdraw from the mechanism, then the expected social welfare at any mixed Nash is at least $\lambda/\max(\mu, 1)$ of the social one.*

Theorem 4. *The PoA of the Relax-and-Round mechanism is $8\eta^2(\eta_2 + 2)$.*

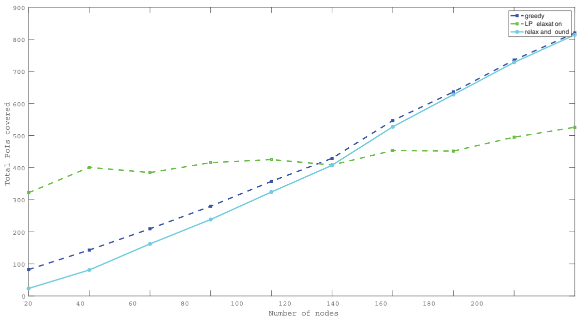

Fig. 3. The total PoIs covered when the density of RSU players increases.

VI. Numerical Results

Our simulation results will be presented in this section. Although we have characterized the worst-case performances of the proposed algorithms in terms of approximation bound on the PoA, we will study the average performance of the proposed LP-based algorithm via simulation results. We randomly deploy a set of RSU candidate nodes in a two-dimensional plane. We evaluate the LP-based algorithm. We compare it with a greedy scheduling method. In the greedy method, we select RSU candidate nodes as follows. Each time we greedily select RSUs as long as the selected RSUs do not conflict with the RSUs already selected. We consider two scenarios with the number of nodes increases. First, we present the average reward of the LP-based algorithm and the average reward of the greedy method when the RSU candidate density in the deployment the plane is fixed. The results are shown in Figure IV-B. Second, we present the average reward of the LP-based algorithm and the average reward of the greedy method when the density in the deployment plane increases. These results are shown in Figure V-B. The results overall clearly show the effectiveness of our algorithms and methodology.

VII. Related Work

Auction theory [6] has been applied to a lot of scenarios in the wireless networking domain, such as crowdsourcing [7], [8], mobile sensing [9], [10], location privacy [11], and routing [12]. On the other hand, there is extensive work on RSU placement. According to what we're informed, there is no existing work on the Nash equilibrium of RSU placement.

VIII. Conclusion

Roadside Unit (RSU) allocation is critical for the functionality and topology control of Vehicular Ad-Hoc Networks. As seen from results and corresponding methodology, we have successfully proposed a mechanism for the RSU allocation problem. We consider the selfish behaviors of RSU allocation and apply a *game theoretic* technique. The mechanism has a small price of anarchy. In relation to potential future work, one open question is to design a mechanism for RSU allocation to minimize the number of RSUs to cover all PoIs. Moreover, the application of the proposed methodology and algorithms to real-world datasets of a varying nature would be an ideal direction to consider.

References

[1] A. Chinnasamy, B. Sivakumar, P. Selvakumari, and A. Suresh, "Minimum connected dominating set based rsu allocation for smartcloud vehicles in vanet," *Cluster Computing*, vol. 22, no. 5, pp. 12 795–12 804, 2019.

[2] Y. Lu, M. Yan, M. Han, Q. Yang, and Y. Zhang, "Privacy preserving feature selection and multiclass classification for horizontally distributed data," *Mathematical Foundations of Computing*, vol. 1, no. 4, pp. 331–348, 2018.

[3] X. Xu, Y. Zhao, D. Zhao, L. Yang, and S. Bakiras, "Distributed real-time data aggregation scheduling in duty-cycled multi-hop sensor networks," in *International Conference on Wireless Algorithms, Systems, and Applications*. Springer, 2019, pp. 432–444.

[4] V. Syrgkanis and E. Tardos, "Composable and efficient mechanisms," in *Proceedings of the forty-fifth annual ACM symposium on Theory of computing*. ACM, 2013, pp. 211–220.

[5] P. Dütting, T. Kesselheim, and E. Tardos, "Algorithms as mechanisms: The price of anarchy of relax-and-round," in *Proceedings of the Sixteenth ACM Conference on Economics and Computation*. ACM, 2015, pp. 187–201.

[6] V. Krishna, *Auction theory*. Academic press, 2009.

[7] A. Singla and A. Krause, "Truthful incentives in crowdsourcing tasks using regret minimization mechanisms," in *Proceedings of the 22nd international conference on World Wide Web*. International World Wide Web Conferences Steering Committee, 2013, pp. 1167–1178.

[8] D. Zhao, X.-Y. Li, and H. Ma, "How to crowdsource tasks truthfully without sacrificing utility: Online incentive mechanisms with budget constraint," in *INFOCOM, 2014 Proceedings IEEE*. IEEE, 2014, pp. 1213–1221.

[9] Z. Feng, Y. Zhu, Q. Zhang, L. M. Ni, and A. V. Vasilakos, "Trac: Truthful auction for location-aware collaborative sensing in mobile crowdsourcing," in *INFOCOM, 2014 Proceedings IEEE*. IEEE, 2014, pp. 1231–1239.

[10] D. Yang, G. Xue, X. Fang, and J. Tang, "Crowdsourcing to smartphones: incentive mechanism design for mobile phone sensing," in *Proceedings of the 18th annual international conference on Mobile computing and networking*. ACM, 2012, pp. 173–184.

[11] D. Yang, X. Fang, and G. Xue, "Truthful incentive mechanisms for k-anonymity location privacy," in *INFOCOM, 2013 Proceedings IEEE*. IEEE, 2013, pp. 2994–3002.

[12] L. Anderegg and S. Eidenbenz, "Ad hoc-vcg: a truthful and cost-efficient routing protocol for mobile ad hoc networks with selfish agents," in *Proceedings of the 9th annual international conference on Mobile computing and networking*. ACM, 2003, pp. 245–259.